Bayesian Modelling of the Wairakei Geothermal Surface Network

ENGSCI 700

Logan Wu

Department of Engineering Science

The University of Auckland

Auckland, New Zealand

lwu308@aucklanduni.ac.nz

*Abstract*—The Wairakei geothermal field is one of the oldest geothermal electricity producers in the world, and it has been instrumental in advancing the utilisation of lower enthalpy fluids. Contact Energy Ltd. is the current operator, and they wish to find ways to increase the productivity of their assets and staff.

Contact’s staff use a system of spreadsheets, test results and real-time measurements to update forecasts of the network and make decisions. Their model has no uncertainty measure.

We implement a Bayesian network model that performs a statistical simulation of the Wairakei surface network.

…

I. Introduction 2

II. Advantage of Bayesian Estimation 2

III. Wairakei Network Structure 3

A. Well Nodes 3

B. Flash Plant Nodes 3

C. Generator Nodes 4

IV. Data Sources 4

A. Network Structure 4

B. Well Test Data 4

C. Live Flow Meters 4

D. Uncertainty 4

V. Data Integration 4

A. Data Extraction 4

B. Pre-Processing 5

C. Automatic and Manual Configuration 5

VI. Simulation Methods 5

A. JAGS 5

B. Sampling Algorithms 5

C. Gibbs Sampling 6

VII. Simulation Implementation 6

A. Model Structure 6

B. Diagnostics 7

C. Verification 7

VIII. Results 7

# Introduction

Contact Energy Ltd., the operator of the Wairakei geothermal field, records a combination of field tests and live data to monitor the state of the geothermal surface network. Staff use the data to make operational decisions about well maintenance, valve positions and long-term sustainability. Currently, the flow of information in this system looks like Figure [REF]. Data from flow meters and well tests is stored for analysis by an operator, who manages the maintenance and operation of the field.

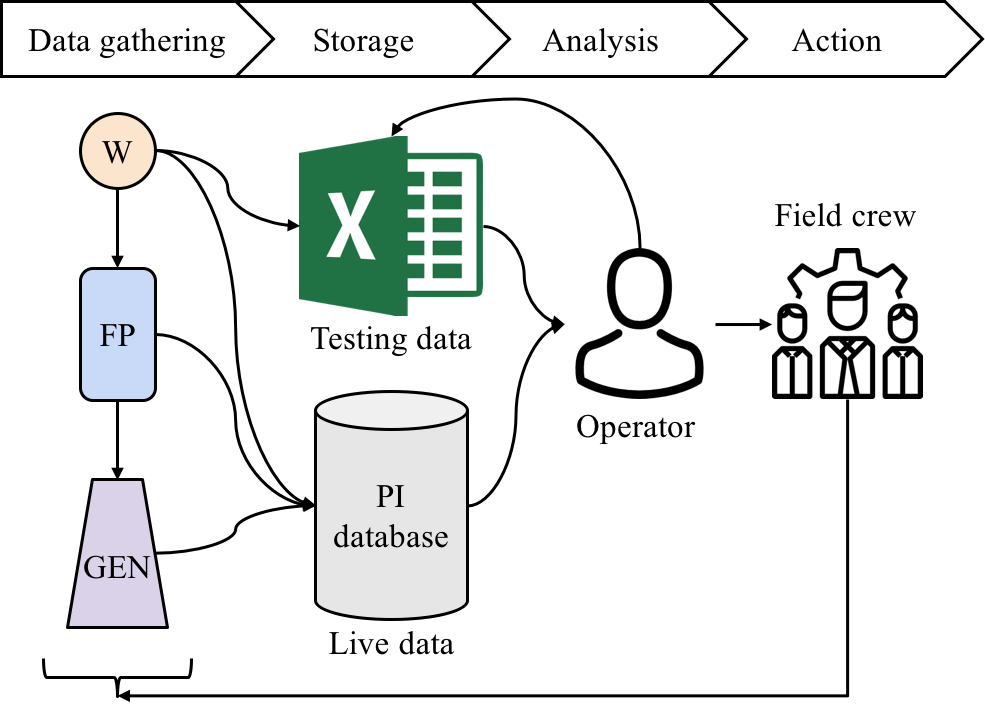


Figure : Current system [CHANGE]

One of the major pain points is between the storage and analysis stages. To access the PI database, the operator exports data into an Excel spreadsheet. They then compare it with the well test data stored in another spreadsheet.

After calibrating the test data, running regressions and making forecasts, they obtain metrics about each well’s condition and make recommendations such as whether to perform a work-over to remove deposits inside the well bore. Accessing and processing the data is a highly manual task, and the sheets have become cumbersome and slow to open. Calibration requires experience to know what the values should look like, adding dependence on a single operator.

In this paper, we develop an intermediate system between the data storage and operator that performs automatic synthesis of the two datasets and assists in the operator’s tasks by automating some of the statistical analysis.

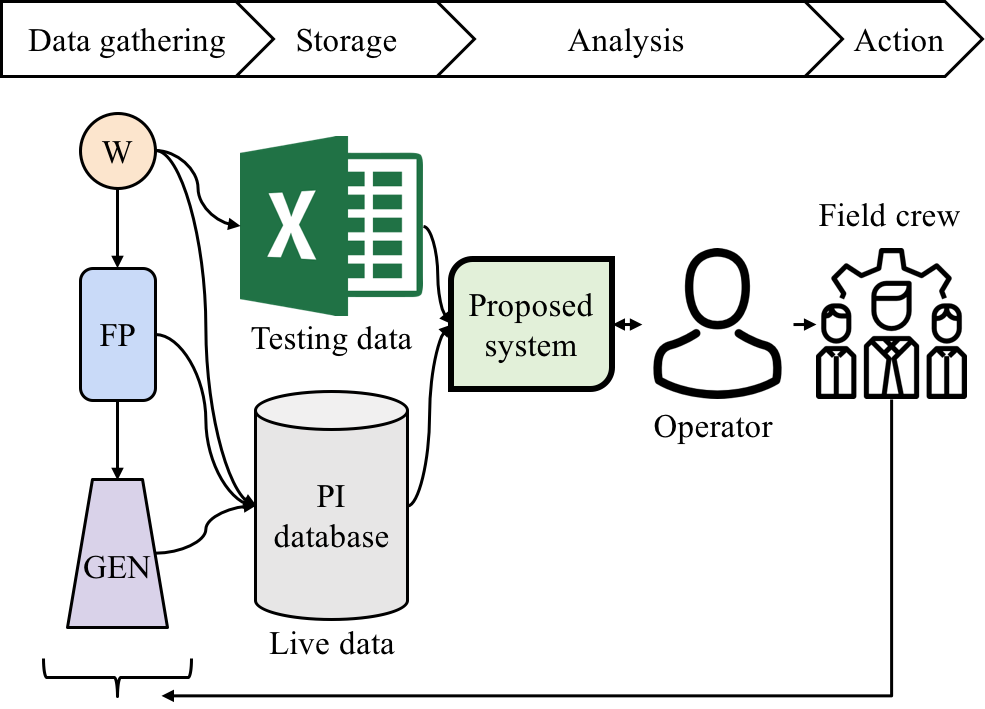


Figure : The location of our product in the proposed system [CHANGE]

There are two advantages to our proposed system:

1. Data from multiple sources is processed into a statistical model that copes with inconsistencies using Bayesian statistics.
2. The operator can interact with the internal model to conduct scenario analysis that includes uncertainty.

# Advantage of Bayesian Estimation

Estimation of uncertainty is an essential part of making informative, yet realistic models. The current geothermal model used by engineers at Contact Energy Ltd. in Wairakei, north of Taupo, is deterministic. It does not take into account factors such as measurement uncertainty and parameter uncertainty when modelling the surface network of wells, pipes, flash plants and power plants. Therefore, there is a lack of understanding around how reliable the forecasts generated by the model are, and how this reliability might change in different parts of the surface network.

A new model that takes into account different sources of uncertainty, from errors in model parameters to errors in measurements, lends itself to Bayesian techniques.

Historically, the main barrier to Bayesian statistics was the computational cost of calculating posterior distributions. In frequentist statistics, the use of approximations allows standard distributions (e.g. the exponential family, notably the Normal distribution) to be used. Many problems with Normal distributions have analytic solutions and are therefore cheap. An example is the Central Limit Theorem, where the average of *n* independent samples converges to a Normal distribution.

With Bayesian statistics, no such assumptions are made. If conjugate distributions such as the Normal-Normal or Gamma-Poisson pairs are chosen for the prior and likelihood, the posterior can be calculated analytically because it is also Normal or Gamma respectively. However, for most other cases there is no closed form of the posterior and it must be estimated computationally. Increases in computing power make sampling possible in a reasonable amount of time.

In a Bayesian simulation, samplers generate stochastic parameter samples from a prior distribution, propagating from the leaves of a directed acyclic graph. Deterministic and/or stochastic operations occur at nodes, which indicate facilities such as wells, flash plants and generators in a geothermal surface network.

The proportional Bayes’ formula is:  
where is the posterior distribution of the parameters *θ* given the data *x*, is the likelihood function of the observed data *x* given *θ*, and is the prior distribution of *θ.* The modeler selects a prior distribution based on their prior beliefs about the parameters. For example:

1. If it is known that , a Beta prior would be appropriate as it is non-zero on the interval [0, 1].
2. If a parameter resides in some ballpark due to expert knowledge, a Normal distribution may be chosen, with a variance determined by the expert’s knowledge.
3. Or, if there is no prior knowledge, this is often represented by a uniform prior, ; any real value is equally likely.

In some cases, these distributions can be chosen using true prior knowledge. Measurement uncertainty is often known to some degree, with some meters rated as having a standard error of 5 units [REF]. Therefore, we set as a Gamma prior on measurement error with a mode of 5. This is the preferred method because it satisfies the Bayes formula where the prior is set prior to observing the data.

The advantage of Bayesian statistics is that the posterior distribution represents the posterior belief of the true network parameters after specifying both the prior beliefs and the data. Frequentist analysis often makes approximations such as the central limit theorem, and does not allow for specification of prior beliefs.

By applying computational Bayesian techniques to the Wairakei geothermal surface network, we will create an algorithmic method to calibrate our model using recorded data, simulate flows in the network and incorporate uncertainty in our predictions.

# Wairakei Network Structure

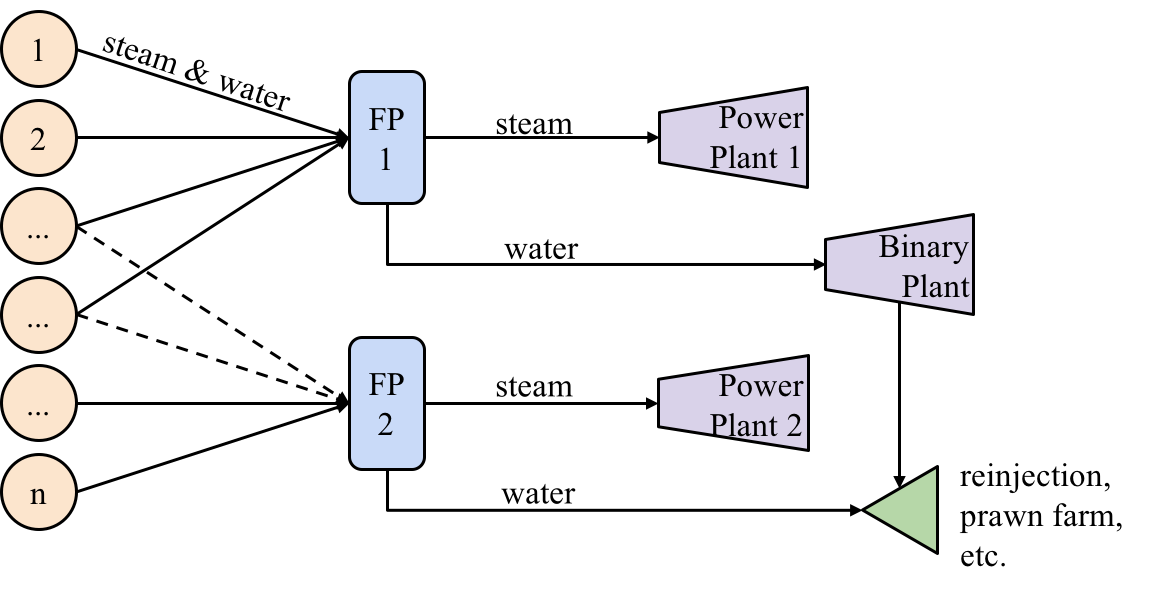


Figure : General structure of the Wairakei surface network.

At a single point in time, the Wairakei surface network can be represented as a directed, acyclic graph shown in Figure [REF]. As some wells have multiple flash plants they can be routed to, the arcs that are enabled are pre-determined according to the configuration we wish to simulate.

There are three types of nodes: wells, flash plants and generators. Each node has inputs, transformations and outputs, and associated parameters.

## Well Nodes

In the model, each well begins with an operating well-head pressure. All active wells generate mixed-phase fluid with the intensive properties of enthalpy *hi* and extensive properties of mass flow rate *ṁi*.

The mass flow can be predicted using bore simulators such as Tough2, but these take a long time to run and it is easier for Contact Energy to run a simpler approximation as . Contact manually approximates *f* using well test data, but they only use three points to fit two degrees of freedom when often they have much more data available.

We use their data set to calibrate a regression model with better support by using all the data. Our regression model also can estimate uncertainty in its parameters.

~~Mass flow at each well is determined as a stochastic function of well-head pressure according to the data:~~

~~where:~~

1. ~~are unknown parameters with uniform priors.~~
2. ~~is a measured well-head pressure.~~
3. *~~t~~* ~~is the number of days since an arbitrary baseline date, January 1~~~~st~~~~, 2000.~~
4. *~~ε~~~~i~~* ~~are normally distributed errors with zero mean and unknown variance~~ *~~σ~~~~2~~*~~, which has a uniform positive prior.~~

~~Independent regression parameters are used for each well, but measurement error variance is shared because the flow measurement error specifications are the same (Section IV.D, Uncertainty).~~

~~By using uninformative priors, the posterior distribution for the parameters of one well is equal to the likelihood function:~~

~~This regression is approximately equivalent to a frequentist ordinary-least-squares regression with errors in the parameters given by a covariance matrix.~~

## Flash Plant Nodes

Elash plants takes flow inputs from a partition of the set of wells, such that every (active) well maps to one flash plant, but more than one well can map to the same flash plant. Configurations can come from historical records, external optimisation routines or any hypothetical setup we wish to test.

At a flash plant *j*, the extensive (mass-flow dependent) properties are summed over its subset of wells *I*(*j*) and intensive (per unit mass-flow) properties are mass flow-weighted averages.

These formulae assume conservation of mass and enthalpy. These assumptions hold if network components are sufficiently sealed and insulated. Enthalpy loss in the Wairakei pipes is estimated at 0.6% in the pipes by [REF], who concludes that it is negligible. *Will need to investigate why the flash plants don’t have enthalpy loss*

Impure steam from the wells causes pitting and corrosion in the generator turbines. Flash plants convert the wet, mixed-phase fluid into pure steam with one or two pressure drops to boil the liquid component of the fluid [REF]. The resulting steam mass flow from a drop to pressure *P* can be calculated by:

where *hf@P* is the specific enthalpy of saturated water at pressure *P* and *hfg@P* is the latent heat of evaporation. The remaining mass flow is the liquid fraction.

The resulting steam and water flows are not constrained to go to the same generator. Many wells send steam to Poihipi and water to the binary plant.

## Generator Nodes

Generators accept intermediate/low pressure steam or water from flash plants, where the subset of flash plants supplying steam to generator *k* is and the subset of flash plants supplying water is . Currently, the power output *Ẇk* from a generator *k* with efficiency *ηk* is proportional to the mass flow of steam feeding it.

or

We are interested in the posterior distribution of total power output, , as this is what we wish to maximise. Analysis of intermediate variables within the network gives insight into where sources of variation or uncertainty in total power output arise.

# Data Sources

We use numerical data supplied by Contact in several forms.

## Network Structure

Contact has provided a schematic indicating the connectivity of wells, to flash plants, to generators. In some cases, wells have in-built flash plants. These are treated the same as any other flash plant, except they only have one well feeding them. When a well has the option to feed to several flash plants, the configuration is pre-determined by any operational decision.

## Well Test Data

Well tests are recorded in an Excel spreadsheet. Bore tests, a specific type of test, are performed at multiple operating pressures. This allows Contact Energy to fit a production curve to the well, as discussed in the Literature Review. The spreadsheet also contains results from Tracer Flow Tests (TFTs), which are easier and cheaper to run because the well can remain connected to the network.

Tests are performed on demand and data is recorded manually.

## Live Flow Meters

Real-time data is supplied using flow meters. However, they do not measure mass flow at the wells, which is why a regression model for well mass flow is created from the well test data.

The benefit of live data is that it is stored once a day in a PI system (treated here as a generic database) in a regular time-series containing every meter. It is therefore much more regular than the well test data. The parameters recorded include well-head pressure, separator pressure, enthalpies and mass flows for facilities excluding wells.

## Uncertainty

To take full advantage of the Bayesian framework, we want to specify well-informed parameters. We have obtained prior estimates for some of the measurement uncertainties by correspondence with Contact Energy:

1. Well test mass flow measurements are ±5%-10%
2. Flash plant mass flow measurements are ±10%
3. Steam to power conversion factors are up to ±5%

Turning these statements into prior specifications is to the modeller’s discretion and will be discussed for the model. However, in practice most sensible priors work if there is enough data available.

# Data Integration

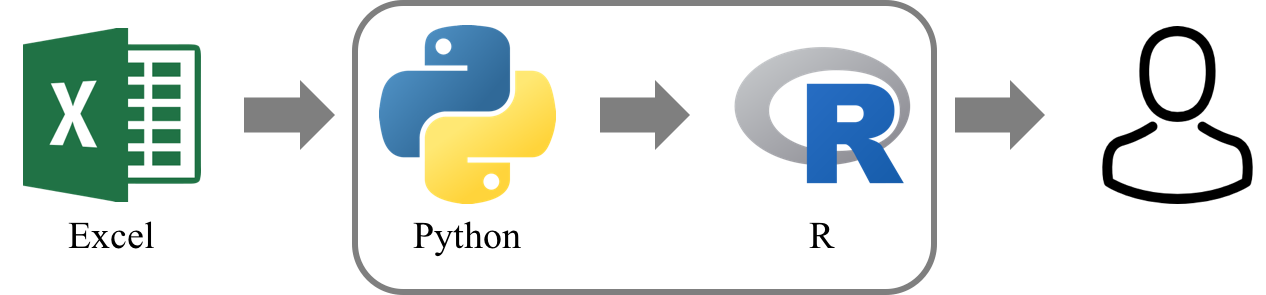


Figure : Workflow for processing the data and running our analysis.

Direct integration of our routine with Contact’s PI systems was not possible for this study because of the commercial sensitivity of live asset data. Exported Excel worksheets therefore provide the main source of data input for our implementation. The section details how we extract the raw data from provided Excel samples, how we process it and what processed data our statistical analysis requires.

## Data Extraction

Accessing the data using the Microsoft Excel desktop application is slow, on the order of ten minutes for the sample data supplied, but hours for the actual operational spreadsheet. Instead, we use a Python script that accepts our unmodified data spreadsheets and immediately converts the data into Pandas DataFrame objects. Without the overhead of Excel and the myriad of formulae within the spreadsheet, loading the data into memory from storage takes seconds.

Our Python routine implements sufficient automatic data-cleaning capabilities that fix known inconsistencies such as capital letters, reject incomplete or erroneous lines, and discriminate between data and meta-data such as comments. Data cleaning takes negligible time.

## Pre-Processing

The original Excel spreadsheets are in human-readable formats. These include well names rather than well IDs, and lacks certain metadata such as the quantities of each facility or the mappings of wells to flash plants.

The second half of the Python script maps facility names to unique integer IDs and converts time formats into the number of days since an arbitrary baseline. These allow the data to be ingested by R.

## Automatic and Manual Configuration

The rest of the workflow is carried out in R. To make the program usable to non-programmers, R reads in configuration options from a separate Excel spreadsheet. Here, the user configures the well mappings and the pressures they intend to run the wells at. The entire network structure can also be changed to test scenarios with different facilities.

An R script reads in the processed data and the configuration file. It uses these to construct instructions for JAGS specifying the stochastic graph’s structure and parameters. R also acts as the interface to JAGS, performing post-processing and visualisation of the outputs.

# Simulation Methods

Sampling methods rely on the law of large numbers. We assume that if we take enough samples from a distribution, we will converge to the true distribution. There are many methods to perform these samples, but often there is a tradeoff between flexibility and efficiency, with Markov Chain Monte-Carlo methods being the most flexible and analytic evaluation being the most efficient but sometimes impossible. We use a specific implementation called JAGS through RJAGS, a package for the R language.

## JAGS

JAGS (Just Another Gibbs Sampler) is a GNU-licensed program that implements a Markov Chain Monte-Carlo method called Gibbs sampling. The exact sub-components of its Gibbs routine are abstracted as a black-box to the user through an R-like syntax.

The statistical model we input into JAGS is a directed, acyclic graph. We then supply JAGS with priors for parameter nodes and data for any nodes with observations.

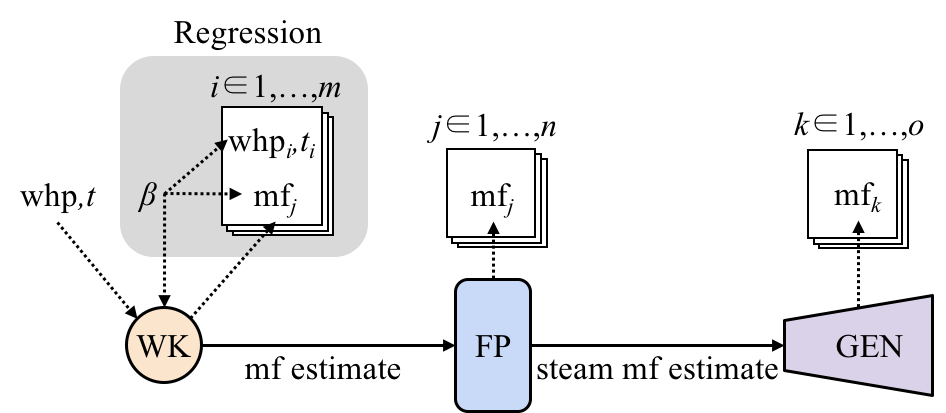


Figure : BUGS-style flow diagram. This is incomplete because the model is too large to show the full hierarchical network.

Figure [REF] shows the information flow between prior nodes, intermediate values and observed values. It is based on a graphical interface from a similar program, WinBUGS, where solid lines are deterministic relationships and dotted lines are stochastic relationships. Root nodes are prior distributions specified by the modeller, and leaf nodes are observations.

In a variation of the Forward-Backward algorithm, JAGS samples from the priors at the root nodes (here, the well-head pressures “whp” and regression parameters *β*) and propagates forward through the arcs. When a parameter value reaches a node we observe through measurements such as mass flow, the likelihood function is computed. The likelihood is passed backward through the network to update the parameters’ posterior , approximated by over repeated sampling.

By computing the posterior distributions for all parameters, we extract the values of the regression coefficients and estimates for flows at any point in the network.

## Sampling Algorithms

At run-time, JAGS automatically chooses the most appropriate algorithm for each node, so different nodes can use different methods. These methods come from separate modules – ‘base’ JAGS and ‘BUGS’.

Both of the following sampling methods are used in a single iteration of a Gibbs Sampler, which will be discussed in the following section.

### BUGS::Conjugate

The conjugate sampler in the BUGS (Bayesian inference Using Gibbs Sampling) module is used when the parameter’s posterior is a conjugate distribution to the prior. Conjugate distributions are where the posterior and the prior come from the same family of distributions. This holds when the prior is the conjugate prior to the likelihood .

The conjugate distributions (priors and posteriors) used in this model are the Normal and Gamma distributions, when the likelihood is a Normal distribution.

Conjugate priors and likelihoods are used wherever possible because the resulting posterior can be calculated analytically. For example, if we have the likelihood of a parameter as where the conjugate prior distributions on mean and variance are and respectively, we can calculate the analytic posterior for the individual parameter assuming the other is fixed:

Note that in JAGS, the Normal distribution is often parameterised by precision rather than variance, leading to a Gamma conjugate prior instead of Inverse-Gamma.

Once the posterior parameters are obtained, samples are obtained by any general method such as the Box-Muller method, an efficient method for transforming independent uniform (pseudo) random variates into standard Normal samples.

### base::Slice

In our model, a Slice Sampler is used for all other parameters in the model that do not use conjugate distributions. The principle of slice sampling treats a univariate density as a uniform bivariate density, with one of the variates giving the same steady-state posterior as the original univariate.

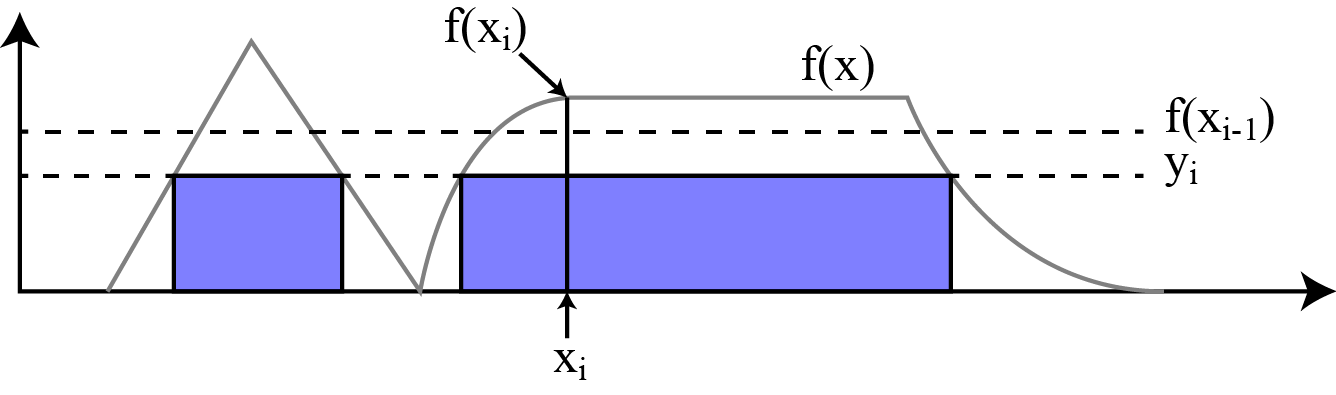


Figure : A slice sampling iteration. The sampling density of xi is shaded in blue.

The steps to take a slice sample are [1]:

Given sample uniformly from .

Given sample uniformly from

The long-run distribution of will converge on . Slice sampling can also be used for discrete variables, but our model only uses continuous parameters.

Slice sampling is a special form of random walker (Metropolis-Hastings) that is relatively simple to implement. By solving the inverse problem for the set , it avoids issues faced by other Metropolis algorithms where the random walker can become trapped by concave functions, as we allow it to jump to any .

## Gibbs Sampling

The previous methods are all implementable as part of a Gibbs sampling algorithm.

A Gibbs sampler is a MCMC (Markov Chain Monte-Carlo) method, where each iteration depends on the state of the chain in the previous iteration in a stochastic function determined by the individual samplers. Our goal is to determine the full joint distribution of all the parameters in our model, which is very difficult. MCMC allows us to construct a Markov chain whose distribution converges on the full joint distribution – if it is ergodic, it will satisfy two conditions:

1. The chain can explore all possible states from any starting state.
2. When run to infinity, its expected state distribution is equal to the joint distribution.

In the Gibbs sampler, each iteration consists of sampling one parameter (or a block of parameters when possible, e.g. multivariate Gaussian) given the most recent values of the other parameters. One iteration proceeds as follows where *i* is the iteration for *p* variables:

This specific MCMC scheme is useful because instead of sampling from a *p*-dimensional posterior, we sample from *p* one-dimensional posteriors.

Although this means each sample is not independent as would be expected from direct sampling routines, its equilibrium state still converges to the correct distribution. The time taken to reach equilibrium depends on how well the random walkers mix within their distributions, which is why conjugate and slice samplers are effective as they do not use a fixed step-size.

The Gibbs sampler can be proven to satisfy the MCMC conditions [REF STATS 731]. JAGS uses the its constructed Markov chain with the univariate conjugate and slice sampling methods to approximate the joint posterior distribution of the geothermal surface network.

# Simulation Implementation

While we run JAGS from R, JAGS code is language-agnostic because it is defined using a text string. We use R instead of similar Python packages because the R interface is better supported.

This section will discuss the components of our JAGS model and the forms of data it requires.

## Model Structure

Despite resembling a procedural coding language, JAGS code is declarative and interpreted simultaneously. However, our code can still be interpreted as a set of ‘steps’ where each block leads into the next.

### Covariate Centering

Our code contains a generalised linear model (GLM) that is incompatible with JAGS’ ‘GLM’ module, a specialised sampler for GLMs that is efficient when there is covariance between the parameters. Since we cannot use the module, we center the covariates. For example:

When there is high covariance, univariate samplers mix poorly because the step of one parameter is highly dependent on the value of another parameter rather than being mostly random. Centering the covariates makes the expectation of covariance zero.

### Well Production Curve Regression

Fluid mass production from the wells has a time-dependent relationship with well-head pressure. Grant and Bixley propose a shifted elliptic form because it has interpretable real-world parameters of a maximum pressure and a maximum mass flow.

Contact Energy’s existing spreadsheets use a centered ellipse, which only requires two data points to fit rather than three for the extra axis shift parameter:

In practice, our model is not trying to estimate maximum pressures and mass flows, since these are theoretical and are not obtained in the field [REF Grant & Bixley]. Therefore, we are not restricted to this form of equation, and we can use a linear regression, which is accurate in the vicinity of the data when the production curve can be approximated as a first-order multivariable Taylor series.

We also incorporate time as a covariate to allow for estimation of the flow decline over time and forecasts into the future. The equation we fit is:

and the corresponding Normal likelihood function per data point (the full likelihood is the product of its components):

where is the mass flow, is a specified well-head pressure, *t* is a specified number of days after a baseline, and is a Normally distributed error of unknown variance. This form has several advantages:

1. Coefficients are interpretable as rates of change.
2. Fewer parameters are less likely to over-fit to small amounts of data.
3. There are no root or power operations. This makes sampling significantly (~5x) faster.

is a product of measurement errors and flow variance. We derive a prior for the measurement error from Contact Energy, and sample from flow variance when we make predictions.

We monitor the traces of the regression parameters to gather statistical estimates for mass flow decline over time and flow variance.

We use a hierarchical Bayes structure to set priors on the regression coefficients , where is precision and is often used because it makes the analytic calculations simpler. Rather than making every individual parameter non-informative, we use the assumption that parameters between wells come from the same (unknown) distribution:

where the hyper-parameters have an non-informative prior

This is physically motivated because:

1. For , pressure loss over time affects multiple wells.
2. For and , wells in a field may have similar production curves.

This is not the same as saying the parameters between the wells are identical. Instead, it fits a distribution of well production curves from which each well is observed.

The biggest benefit of this added bias is the reduction in variance for wells with insufficient or no data – we instead make an educated imputation that their production curve is similar to the wells we have data for, rather than having absolutely no imputation at all.

Our regression model is supported by the diagnostic plots of observed mass flows against fitted mass flows, and a standardised residual plot using residuals:

where is sampled from the regression.

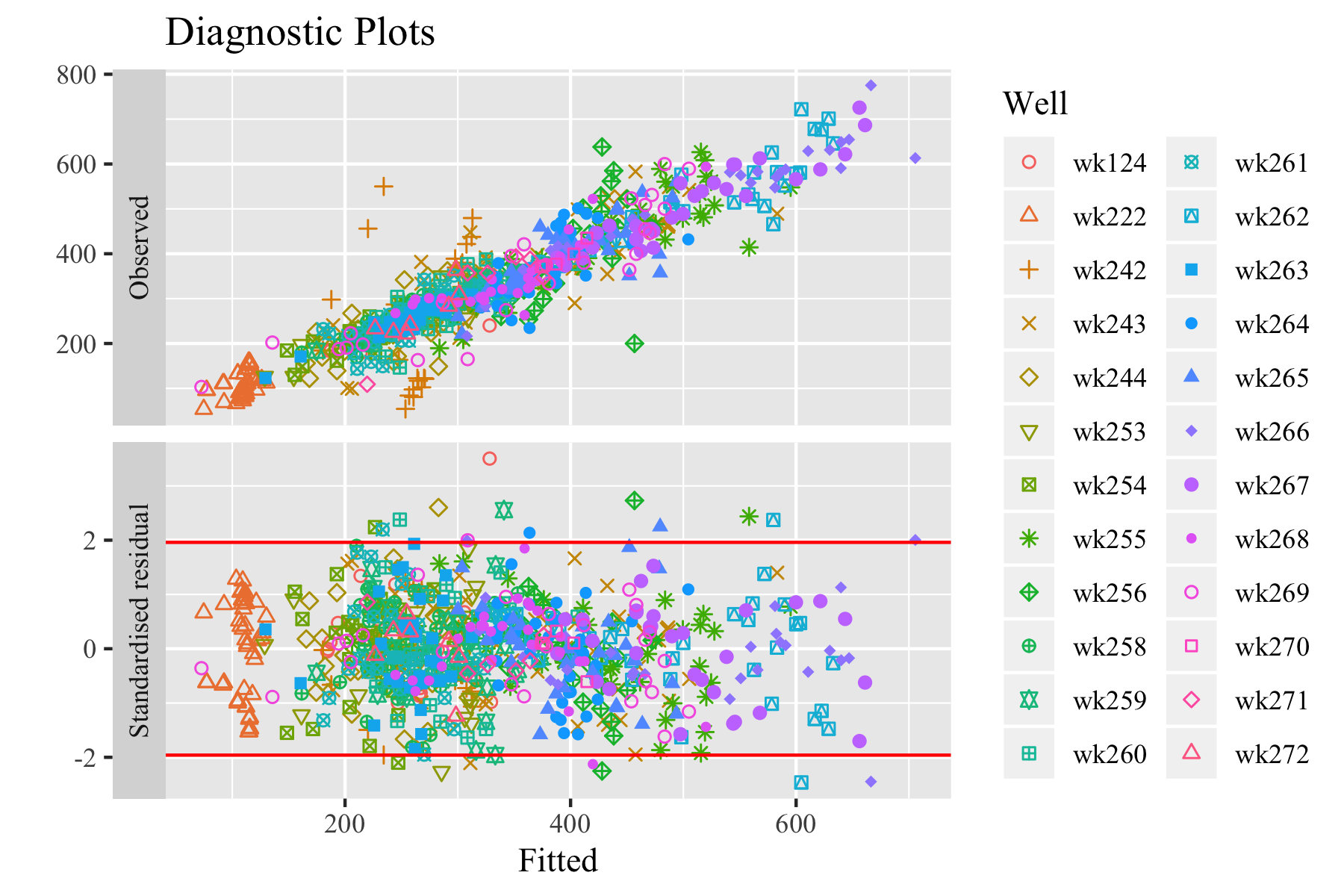


Figure 7: Diagnostic plots for the liquid wells regression. We expect 95% of standardised residuals to be within the red lines with no trend.

A well-fitting regression shows a linear correlation between the fitted and observed values. If we believe the residuals are Normally distributed, the standardised residual plot will have a standard Normal distribution for all the fitted values. Both of these qualities are displayed in Figure [REF].

### Prediction

With posterior production curves fitted for every well, we can estimate the mass flows at a given well-head pressure and date, specified in the configuration file. We also assign a measured enthalpy to the well flows, or apply a hierarchical posterior to any missing enthalpy values.

Network flows can now propagate through the network to flash plants, dummy generators and generators.

### Flash Plant Flows

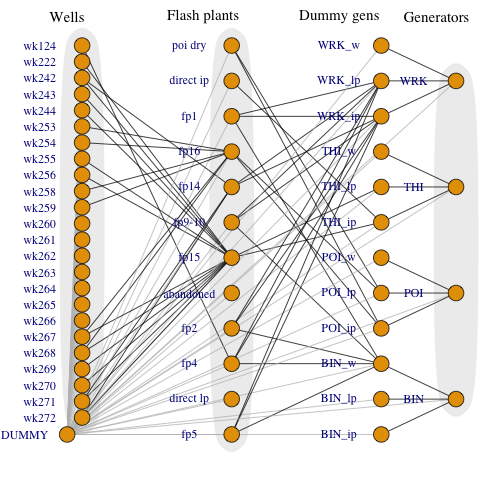


Figure : Actual network connectivity implemented with just the liquid wells. Dummy arcs are in grey.

Well-to-flash-plant assignment is specified in the configuration spreadsheet. To calculate mass flows and flow enthalpies entering a facility (both flash plants and generators), we use the formulae in Section III.B. During preprocessing, we add a dummy node to all *I* to ensure that *I*(*j*) is not an empty set; otherwise, JAGS throws an index error.

Generating the sets *I* is the main reason why facilities need to be re-identified using a unique integer rather than a string – despite using an R-like syntax, JAGS only accepts integer indices.

### Generator Flows and Power Conversions

In pre-processing, generators have three dummy generators placed in the network before the actual aggregated power is calculated. Each generator has a dummy for each type of fluid flow – intermediate pressure (IP), low pressure (LP), or water (W). Whether these are actually used depends on the configuration inputs. For instance, the binary *BIN* plant will only ever use the *BIN\_w* dummy because in reality, it is a waste to send steam to the low-efficiency binary generator.

Dummy generators flow into their respective generator. Contact Energy calculates power output as a function of the bulk mass flow in Section III.C. These efficiencies are given to us in units of Tonnes/day/MW. Our uncertainty in the conversion factor is ±5%. We interpret this as , which holds for small (<10%) percentages.

## Monitoring

Setting a monitor is how JAGS returns outputs to the user. There are three types of monitors:

1. Trace, showing every sample of a parameter.
2. Mean/(co)variance, showing the cumulative current mean/variance of a parameter up to and including a sample.
3. DIC (deviance information criterion), to evaluate goodness of fit of the model to the data.

We are not interested in mean and variance because the same information can be extracted from the trace. DIC is also not useful because it returns a single number for the entire model for comparison purposes. Since our data is intended to be online, DIC cannot be compared between runs. Also, graphical goodness of fit techniques are preferred because the user can identify specific issues that would not be possible with a single value, such as erroneous data.

The first monitor we set is on the well regression. Rather than monitoring the mean and covariance of the regression parameters, it is easier to make predictions and plot these against the original data points in Figure [REF]. We see a line that is close to the 2017 data and a reasonable thickness as an estimate of the error.

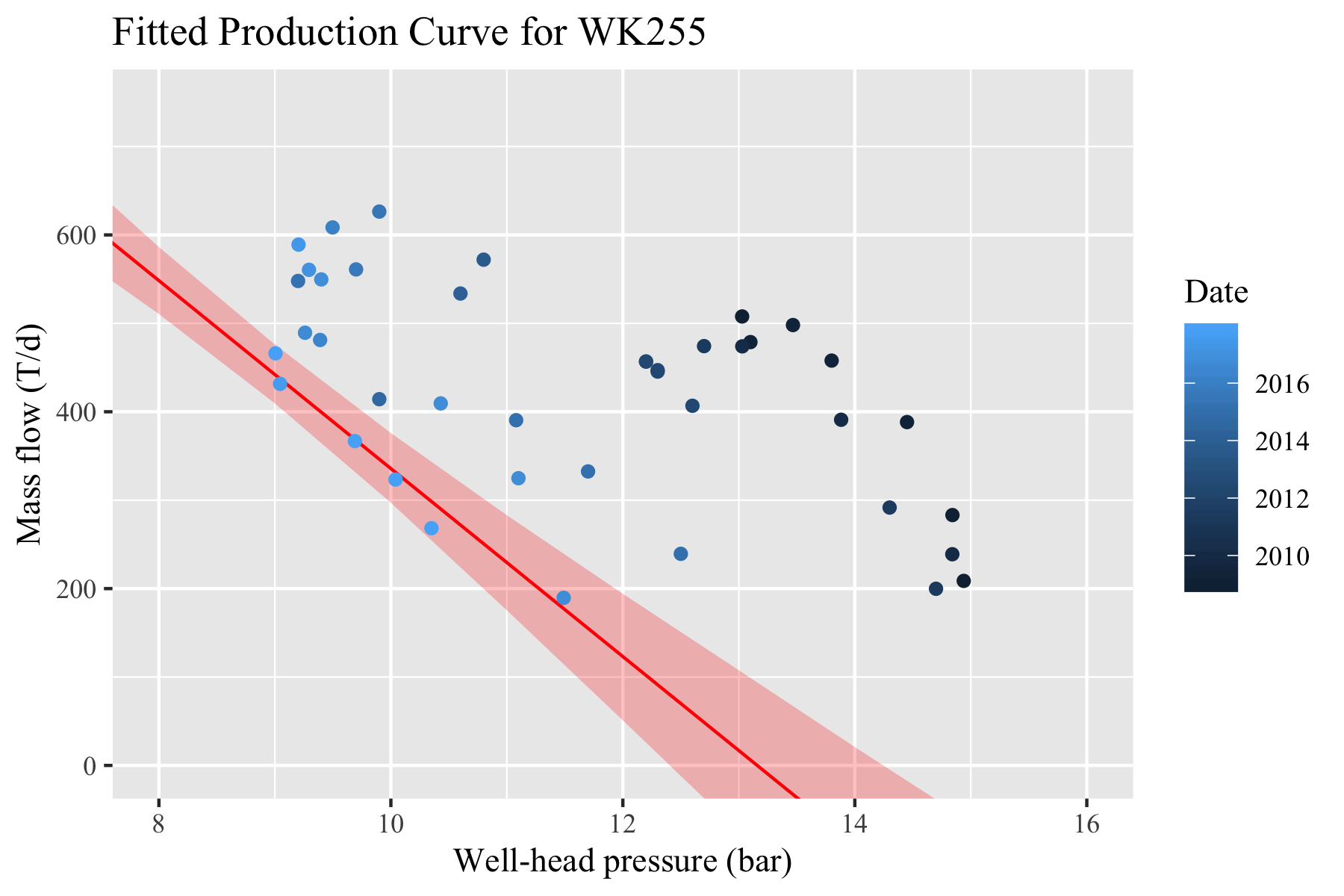


Figure : Red line is the mean prediction for 2018. Shaded region is the 95% confidence band.

Next, we monitor mass flows at each facility and their probability densities. The uncertainty in the parameter is represented by the width of its distribution; conversely, the precision can be interpreted as the height of its peak because densities integrate to one.

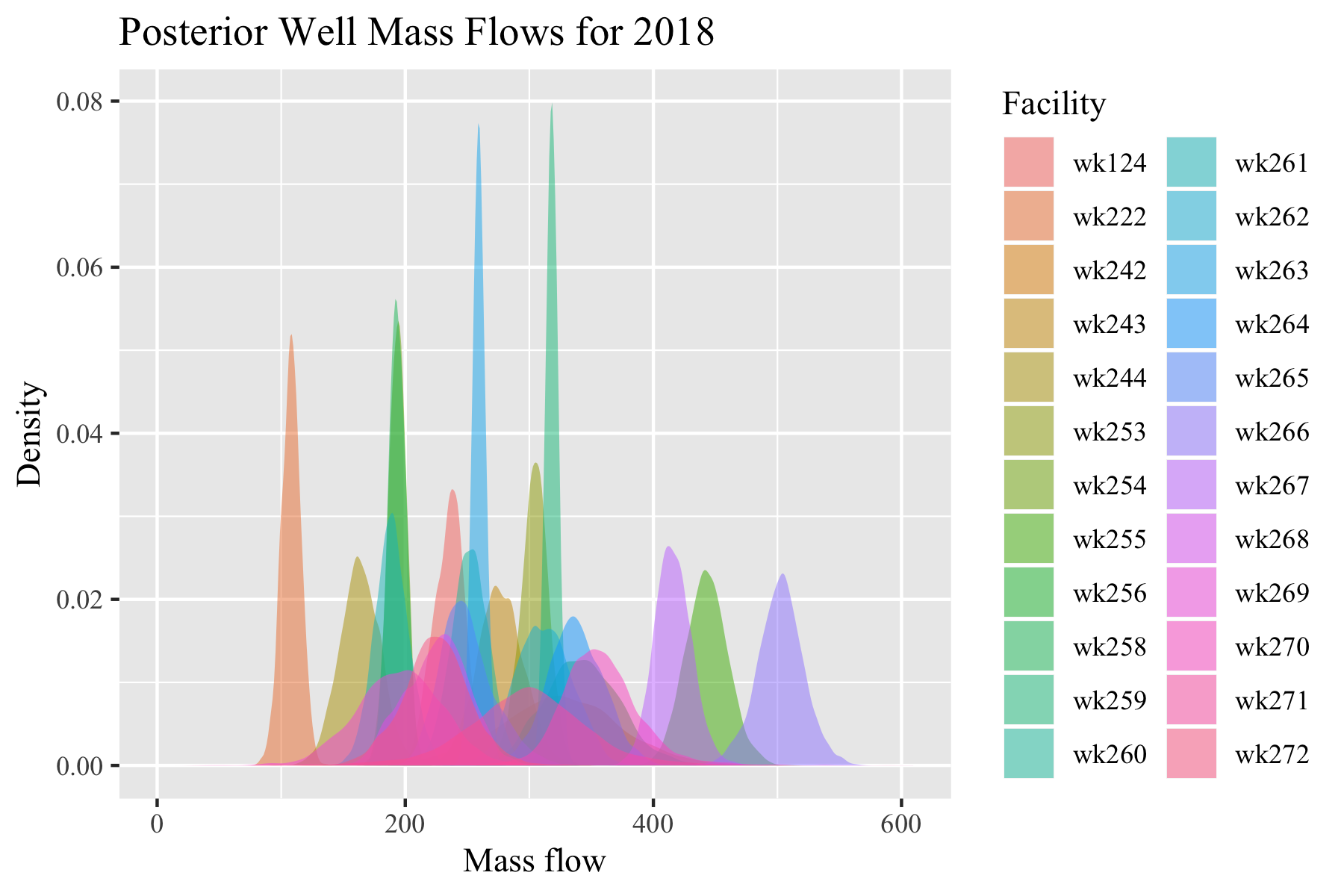


Figure : Mass flow estimates for liquid wells

Figure [REF] presents the posterior densities for mass flow at selected wells. We can see that some wells have more variation than others, and this is due to several reasons:

1. Lack of data to fit the regression.
2. Highly variable mass flow in some wells.
3. Nonlinearity in the mass flow (i.e. bad fit).

These estimates for variation are the strength of Bayesian inference; we can now sample from the posteriors and propagate our uncertainty through the network.

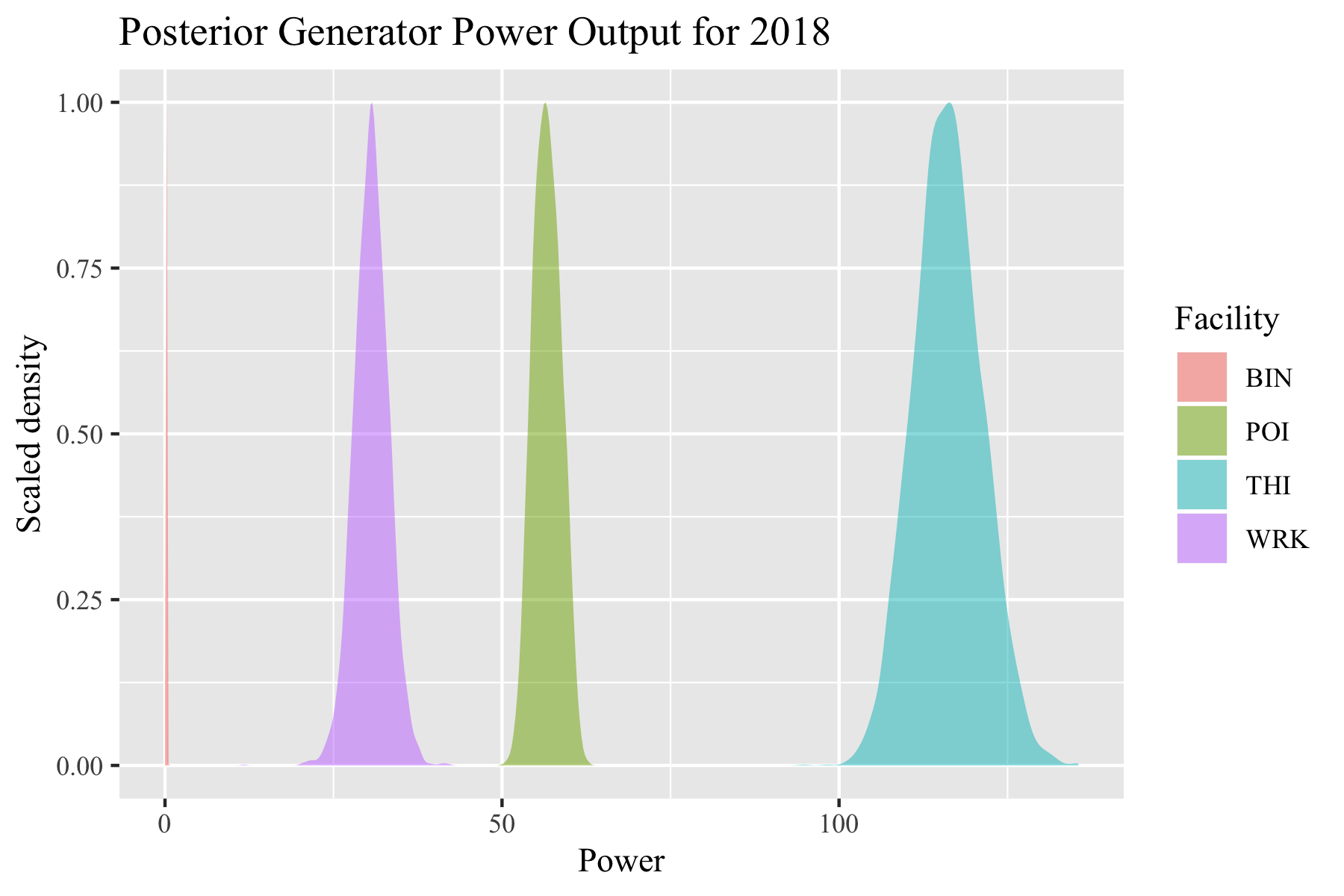
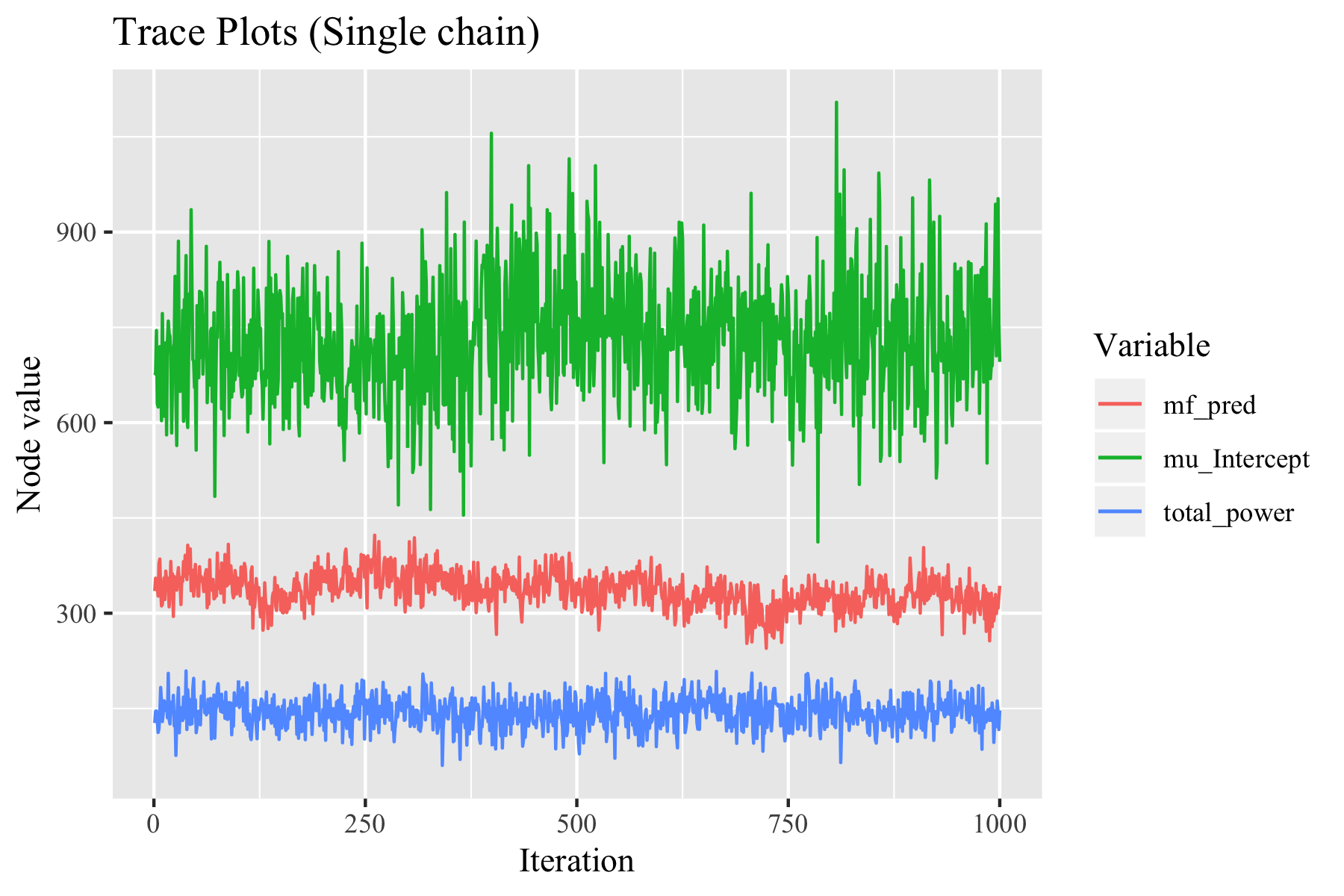


Figure : Scaled posterior estimates for power output per station, given the liquid wells.

Figure [REF] shows how those uncertainties have propagated all the way through to the final power estimates. The widths of the distributions are due to uncertainty in the mass flows and the power conversion factor.

## Diagnostics

One of the difficulties with MCMC approximations is they often require a burn-in (warm-up) period before settling into the stationary distribution of the chain. Only the stationary distribution corresponds to the joint distribution we are interested in. In most practical uses, there is no way to predict convergence, so it must be done by monitoring the sample trace and running diagnostic tests.



Poor convergence and mixing is represented by a strong trend at the beginning of the trace plot. This is not present in any parameters in Figure [FIGREF], but visual analysis of many traces is impractical.

The implementation of Gibbs Sampling we use, JAGS (Just Another Gibbs Sampler), also provides diagnostic tests in the CODA package. There are two main tests to confirm this:

### Geweke

Geweke’s convergence diagnostic for MCMC samples tests for equality of the means in the first 10% and last 50% of the trace (the samples in iteration order). The means will be equal if the sample is drawn from a stationary distribution, indicating the burn-in period has been successfully excluded. Geweke’s statistic has a T-distribution using the following T-test statistic:

where and are the sample means of the first 10% and last 50% of the samples, *s2* are their corresponding standard variances, and *n* and *m* are the number of samples in the two groups. Spectral densities are used to estimate the sample variances. [REF].

Most of the parameters pass Geweke’s test with a z-score (normal approximation of the T-statistic) less than 1.96 for a 95% confidence interval. However, the changes in the power output traces were significant, so a short burn-in of 200 iterations was introduced.

### Gelman [REF]

Gelman’s test gives the potential scale reduction factor for each parameter. This requires at least two parallel chains using independent random variates (JAGS uses the Wichmann-Hill, Marsaglia-Multicarry, Super-Duper and Mersenne-Twister pseudorandom generators for the first four chains to ensure they are independent), and tests whether the chains have converged to identical distributions. If the chains have not converged, the scale reduction factors will have upper confidence limits greater than one and the samples obtained are likely to be over-dispersed.[REF]

Executing Gelman’s test on all monitored parameters runs into issues with an internal Cholesky matrix factorisation of an ill-conditioned matrix. Testing a smaller selection yields:

INSERT TABLE and do analysis

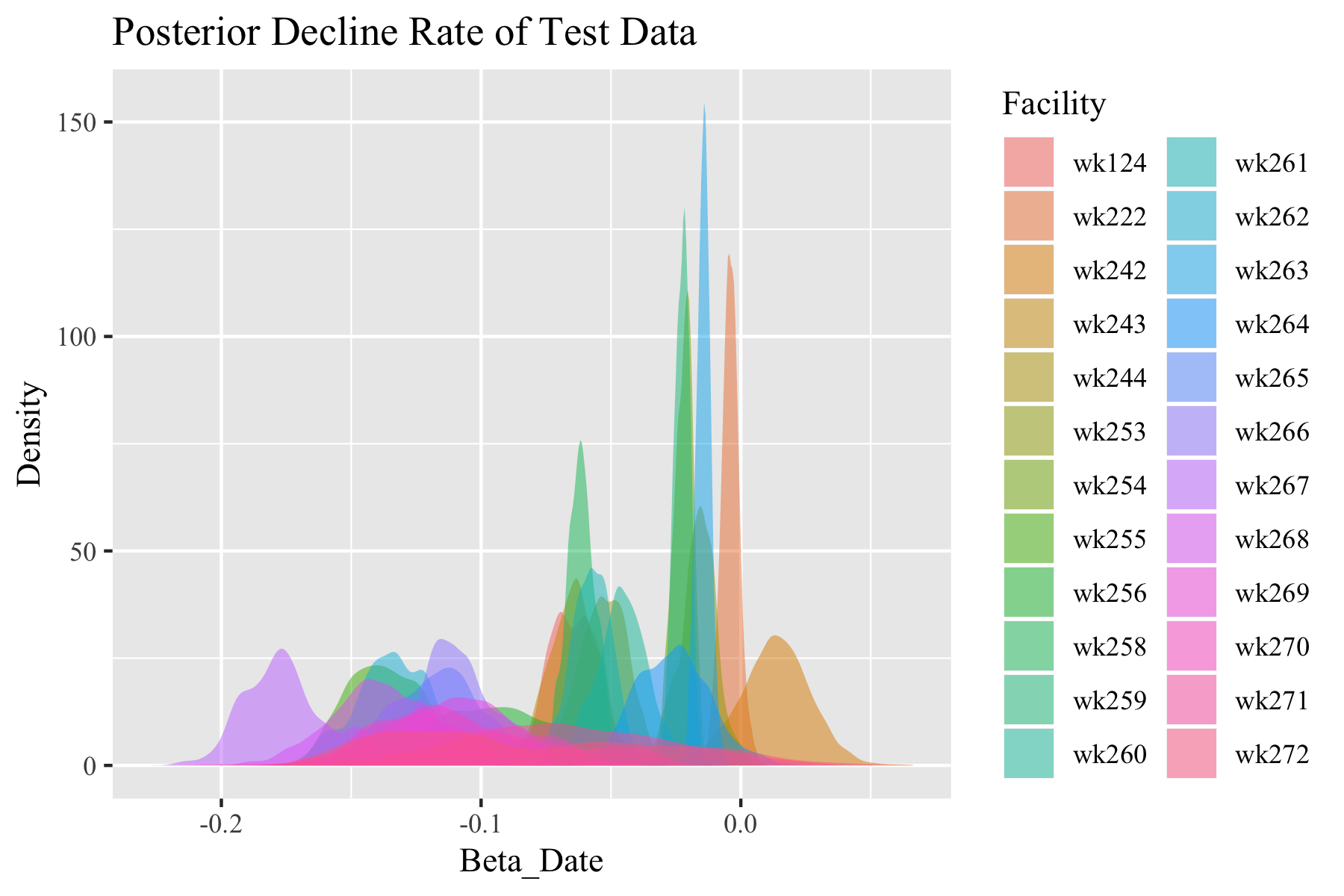
Some of the upper CIs are greater than one, but not by much. Running the simulation for longer will shrink the CI.

*TODO: Confirm simulation passes both tests and others (Raftery, autocorrelation).*

# Results

We analysed the simulation traces using R.

## Individual well declines



Over the recorded data wells have declines between zero and -0.2 T/d for a fixed well-head pressure.

## Flow variances

# Conclusions

# Future Work

##### References